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MODIFIED WHITTAKER TRANSFORM ON $\mathcal{L}_{\nu,r}$-SPACE

Yuri V. Vasil'ev

Abstract

The paper is devoted to the study of the integral transform

$$[\mathbf{W}_{\rho,\gamma;\sigma,\varpi;a,b,\lambda}f](x) = x^{\sigma}\int_0^{\infty} e^{-\frac{\lambda}{2}x^a t^b} W_{\rho,\gamma}(\lambda x^a t^b) t^{\varpi} f(t)dt \ (x>0)$$

$$(\sigma,\varpi \in \mathbf{C};\ \lambda,a,b>0;\ \rho,\gamma \in \mathbf{C})$$

with the Whittaker function $W_{\rho,\gamma}(z)$ in the kernel on the space $\mathcal{L}_{\nu,r}$ ($\nu \in \mathbf{R} = (-\infty,\infty),\ 1 \le r \le \infty$) of Lebesgue measurable functions f on $(0,\infty)$ such that

$$\int_0^{\infty} |t^{\nu} f(t)|^r \frac{dt}{t} < \infty \ (1 \le r < \infty), \quad ess\ sup_{x>0}[x^{\nu}|f(x)|] < \infty \ \ (r=\infty).$$

Mapping properties such as the boundedness, the representation and the range of the transform $W_{\rho,\gamma;\sigma,\varpi;a,b,\lambda}$ are given on the space $\mathcal{L}_{\nu,r}$.

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